Effect of Friction on Coupled Contact in a Twisted Wire Cable

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A strand or cable consists of a central core surrounded by a number of wires wound helically in a single layer or multilayers. There are three modes of contact in a simple straight strand. The first type is a core-wire contact in which the wires in the layer are in contact with the core only. In the second type, the wires in the layer are in contact among themselves and not with the core, while in the third type there is a coupled contact among the core and all the wires. Most literature handled the cable assembly with either the core-wire or the wire-wire contact because of the simplicity of the loads acting in these distinct contact modes. An attempt is made in this paper to model the strand with a coupled core-wire and wire-wire contact and deduce its equations of equilibrium. The numerical analyses of strand force, twisting moment, strand stiffness, contact force, and contact stress are carried out based on the equilibrium of thin rods and the results are compared with earlier research works. The importance of the inclusion of interface forces and their effects is studied.
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1 Introduction

The importance of cables is very vital in almost all fields of technology from electrical power transmission to ship towing. Stranded cable geometry may lead to core-wire or wire-wire or coupled core-wire and wire-wire contacts depending on the construction of the strand and the type of loading. The contact modes may change from one to the other depending on the loads and the deformation of the core and wires, especially when they are made with different materials. Most literature analyzes the strand with either the core-wire or the wire-wire contact, which leads to the interpretation of results due to simplified assumptions.

The analysis of the cable in a coupled core-wire and wire-wire contact is essential to understand the importance of the interfacial loads and their effects. Furthermore, few researchers have considered the effect of friction at the interfaces, which lead to a wider variation in the results. The effect of tangential distributed forces and normal distributed forces on wire-wire contact and the normal distributed forces on core-wire contact is considered [1]. The mathematical model for a seven wire strand is developed by considering the effect of friction between core and wire and the theoretical model is compared with the experimental results [2,3]. The response of a strand due to axial, bending, and torsional forces is investigated considering the effect of friction between core and wire [4]. A finite element model for core-wire contact is developed considering the friction and material plasticity effects, however, the effect of tangential distributed forces between core and wire is not considered [5]. The quantity of energy dissipated in a cable is investigated due to the frictional dissipation at the core-wire interface due to axial loading of the strand [6]. The analytical expression for the maximum contact stress induced at the core-wire interface is obtained [7]. The effect of bending and torsional stiffness of the individual wires on the strand is analyzed [8]. The symmetric linear elastic model for a cable with a rigid core using discrete thin rod theory is presented [9]. The authors of Refs. [2–9] considered only the effect of normal distributed force in the wire, neglected the binormal and tangential distributed forces and moments in the wire. A mathematical model to represent the effect of tangential and normal distributed forces in a coupled contact is formulated [10].

The aim of the present work is to refine the mathematical model as proposed by Parthasarathy and Sathikh [10] and find out the response of the strand considering all the forces in a coupled contact with fixed end conditions.

2 Extraction of the Model

In the following analysis, each wire is considered in the strand as a long slender curved rod. Figures 1(*a*) and 1(*b*) depict the cross section and the developed geometry of a seven wire strand in the coupled contact. The forces and moments in the helical wires act along normal, binormal, and tangential directions, as shown in Fig. 2. The components of the force resultant acting on the cross section of the wire are denoted by T, N, N' and the components of moment acting on the cross section are denoted by H, G, G'. The components of the distributed force per unit length of the wire are X, Y, and Z and the components of the distributed moment per unit length of the wire are K, K', and Θ. Along any line of contact between the helical wires, there exists the normal distributed force U and the tangential distributed forces V and W, as shown in Fig. 3.

Along the line of contact between the core and helical wire, the normal distributed force S and the tangential distributed forces P and Q act as shown in Fig. 3.

The above interfacial forces can be related to the forces and moments in the wire as under

$$X = -2U\cos\beta - S, \quad Y = 2V\cos\beta \pm P, \quad Z = Q \tag{1}$$

$$K = 2WR_w \sin\beta, \quad K' = QR_w, \quad \Theta = 2VR_w \pm PR_w \tag{2}$$

where R_w is the radius of the wire and β is the contact angle, which locates the lines of action of the line contact loads U on a wire due to its neighbors.

The equations of equilibrium of all the forces and the moments acting on an infinitesimal element of the wire are obtained from the theory of slender curved rods [11]. Since the strand is considered long, all the derivatives of stress resultants and moments with respect to the arc length of the wire are neglected. Since the helical wire is wound on a straight cylindrical core, the normal curvature and the associated normal bending moment of the wire are zero. Hence, the equations of equilibrium are written as

$$-N'\tau + T\kappa' + X = 0 \tag{3}$$

$$N\tau + Y = 0 \tag{4}$$

$$-N\kappa' + Z = 0 \tag{5}$$

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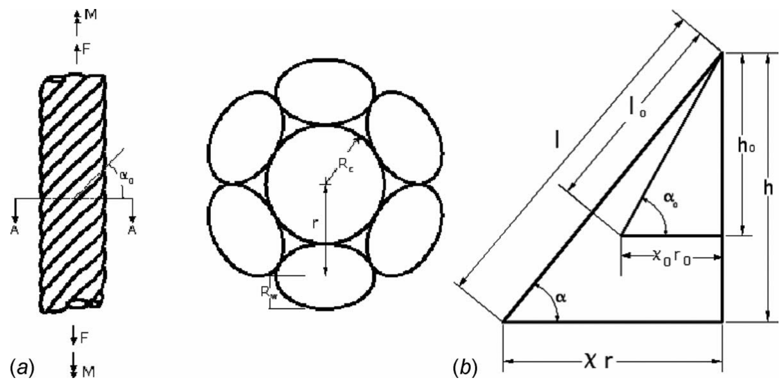

Fig. 1 Strand geometry

$$-G'\tau + H\kappa' - N' + K = 0 \tag{6}$$

$$N + K' = 0 \tag{7}$$

$$\Theta = 0 \tag{8}$$

where κ' is the binormal curvature and τ is the twist of the wire.

When the slipping between helical wires during the extension of the strand is considered, the tangential distributed forces between the core-wire and the wire-wire are given by

$$V = \mu U; \quad W = \mu U; \quad P = \mu S; \quad Q = \mu S \tag{9}$$

where μ is the coefficient of friction between the core wire and the helical wire.

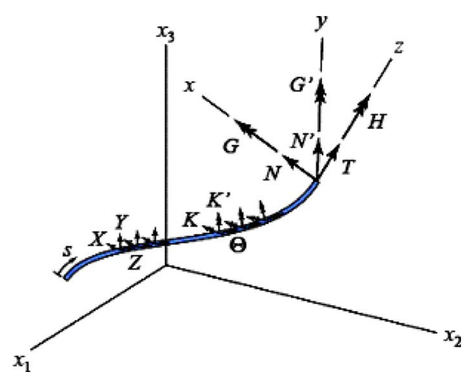

Fig. 2 Forces and moments on a helical wire

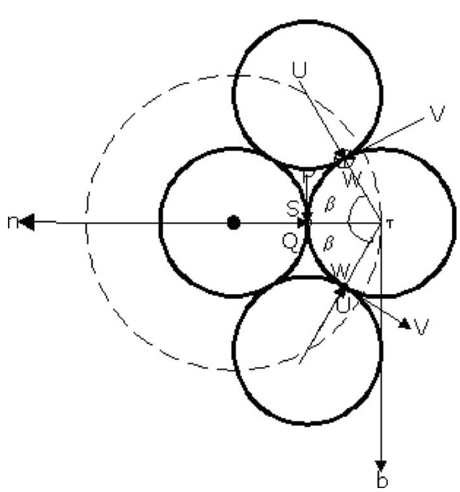

Fig. 3 Distributed loads on a helical wire

On substitution and rearranging, the following equations are obtained:

$$N' = \frac{-G'(R_w\tau^2 + R_w\tau\kappa' + \tau) + H(R_w\kappa' + R_w\tau\kappa' + \kappa') - TR_w\kappa' \sin\beta}{(\mu R_w\tau\sin\beta) + (R_w\tau) + (R_w\kappa') + 1} \tag{10}$$

$$S = T\kappa' - N'\left(\tau + \frac{1}{\mu R_w \tan\beta}\right) - G'\left(\frac{\tau}{\mu R_w\tan\beta}\right) + H\left(\frac{\kappa'}{\mu R_w\tan\beta}\right) \tag{11}$$

$$U = \frac{G'\tau - H\kappa' + N'}{2\mu R_w \sin\beta} \tag{12}$$

The resultant axial force and axial twisting moment acting on the outer layer of the strand is given by

$$F_w = m[T\sin\alpha + N'\cos\alpha] \tag{13}$$

$$M_w = m[(H\sin\alpha) + (G'\cos\alpha) + (Tr\cos\alpha) - (N'r\sin\alpha)] \tag{14}$$

where "m" is the number of wires in that layer.

The central core is under the action of axial forces, twisting moments, and lateral forces along the line of contacts. To evaluate the deformation of the central core, the line forces S on the curved surface of the core are replaced by a statically equivalent uniformly distributed lateral pressure with intensity

$$q = \frac{mS\cos\alpha}{2\pi R_c} \tag{15}$$

where R_c is the radius of the core and α is the helix angle of the wire.

Therefore, axial strain of the core is given by

$$\varepsilon = \frac{1}{E\pi R_c}\left[\frac{F_c}{R_c} + \nu mS\cos\alpha\right] \tag{16}$$

where F_c is the core axial force and ν is the Poisson's ratio of the material of the core.

Similarly, the transverse strain of the core is given by

$$\varepsilon_{tc} = \frac{1}{E\pi R_c}\left[\frac{mS\cos\alpha}{2}(1-\nu) + \frac{\nu F_c}{R_c}\right] \tag{17}$$

By solving Eq. (16) and (17), the core axial force becomes

$$F_c = \frac{2E\pi R_c^2}{(1-\nu)}\left[\left(\frac{(1-\nu)\varepsilon}{2} - \nu\varepsilon_{tc}\right)\right] \tag{18}$$

The transverse strain of the core is also given by

$$\varepsilon_{tc} = \frac{R_c(1 - \nu\varepsilon)}{R_c} - 1 \tag{19}$$

The twisting moment of the core is determined by the equation for torsion of a shaft with circular cross section and is given by

$$M_c = \frac{E\pi R_c^4}{4(1 + \nu)}\tau_s \tag{20}$$

where τ_s is the angle of twist per unit length of the core.

The strand axial force and the twisting moment are expressed from the respective core and wire loads and given by

$$F = F_c + F_w \tag{21}$$

$$M = M_c + M_w \tag{22}$$

The above expression can also be expressed in terms of strand axial strain ε and strand rotational strain ϕ as under

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} F_\varepsilon & F_\phi \\ M_\varepsilon & M_\phi \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \phi \end{Bmatrix} \tag{23}$$

where F_ε, F_ϕ, M_ε, and M_ϕ are the stiffness constants of the strand.

Depending on the end condition of the strand, the above constants can be expressed appropriately. In the case of fixed ends, i.e., the strand rotational strain $\phi=0$, the axial and torsional stiffness constant of the strand are given by

$$F_\varepsilon = \frac{F}{\varepsilon} \tag{24}$$

$$M_\varepsilon = \frac{M}{\varepsilon} \tag{25}$$

In a line contact, the stress distribution is complex. The effect of stress concentration is considered and contact stress is determined theoretically using the Hertz solution. Along these lines of contact, the movement of the wires occur depending on the resisting the friction force that is developed at the interface. The resultant force due to contact is the coupled contact mode, which was given by

$$C_f = (2U \cos \beta) + S \tag{26}$$

From the Hertz contact stress theory, the maximum normal contact stress is given by

$$\sigma_{nc} = \frac{b}{\Delta} \tag{27}$$

where $b=\sqrt{2C_f\Delta/\pi}$, $\Delta=4(1\text{-}\nu^2)/[1/\rho_c+1/R_w]E$, and $\rho_c=R_c/\sin \alpha$.

2.1 Numerical Example. A cable comprising six wires wound around a central core is considered to ensure a coupled contact arrangement. The data pertinent to this cable are six wires, helix angle of 75 deg, material of core and helical wires are steel, Young's modulus elasticity of 2×10^5 N/mm^2, Poisson's ratio of 0.3, friction coefficient of 0.5, core radius of 5.129 mm, and helical wire radius of 4.871 mm.

2.2 Results and Discussion. Using the numerical data of the strand given above, the results of strand force, strand twisting moment, strand stiffness, contact force, and contact stress are obtained for the fixed end condition. In the fixed end condition, there is no rotational strain on the strand. The models [4,6–9] are worked with the above cable data and the results are compared with the present model. The variations in the above results are shown as a function of strand strain in Figs. 4–9. It is found that the coupled contact mode exists only in the initial stage of loading, i.e., up to a strain level of 0.0018 and thereafter turns into core wire contact. The figures are plotted only up to the range where coupled contact exists. Beyond this range, contact pressure between the wires is always negative indicating that during deformation of the strand, the constraint of central core always causes separation between helical wires. For all the models, analysis is done until the wires are not separated. The strand force, strand stiffness, and contact stress are higher in the present model compared with other models because all the wire forces are considered along with the effect of friction.

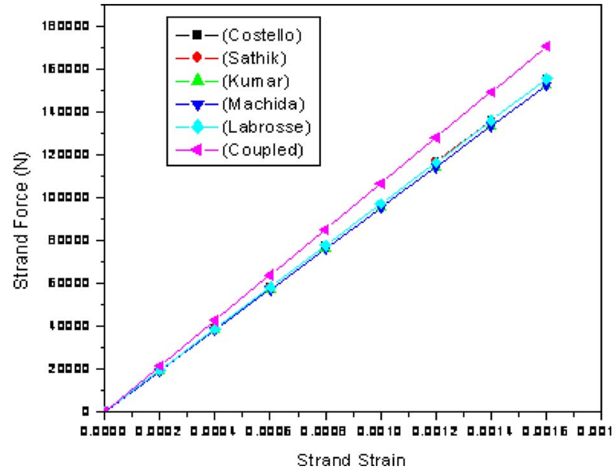

Fig. 4 Strand force

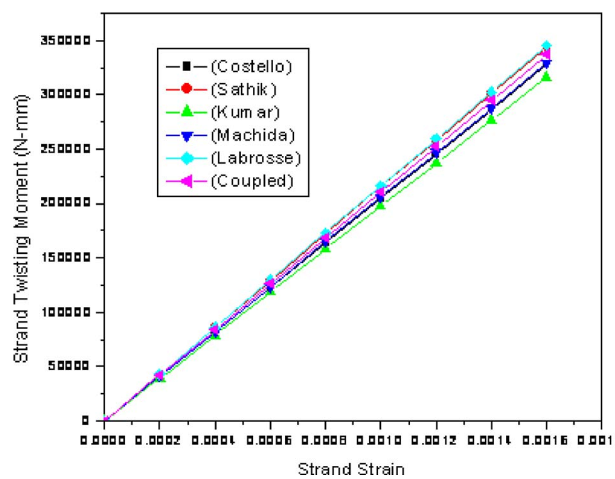

Fig. 5 Strand twisting moment

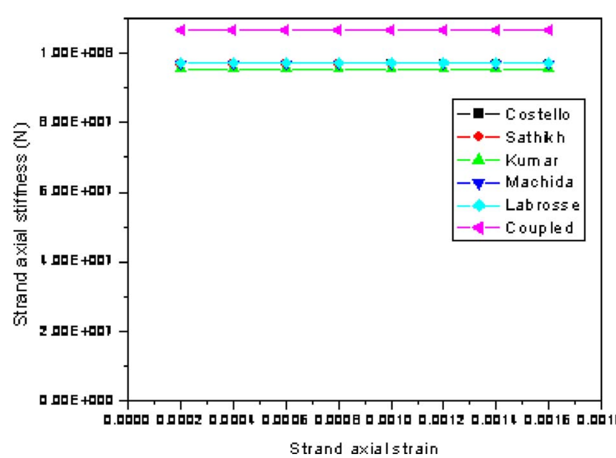

Fig. 6 Strand axial stiffness

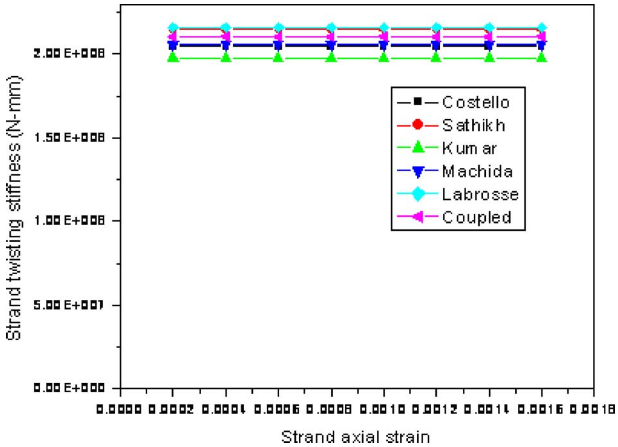

Fig. 7 Strand twisting stiffness

3 Conclusion

In a helical wired strand, the nature of the contact between the wires and the core play a significant role in predicting the behavior of the strand. A strand with a core-wire and wire-wire coupled contact is analyzed. During the initial stages of loading, the coupled contact persists, thereby, all the interface contact loads influence the strand stiffness. Hence, a higher stiffness is predicted. During further extension of the strand, the deformation in the wires causes separation between the wires leaving the strand under the core-wire contact mode only. Although the coupled (core-wire and the wire-wire) contact mode exists during the initial stages of loading, its significance is relevant to predict the strand behavior in this stage. It is hoped that this influence will be noted by the design engineer in order to design the cable geometry suitably.

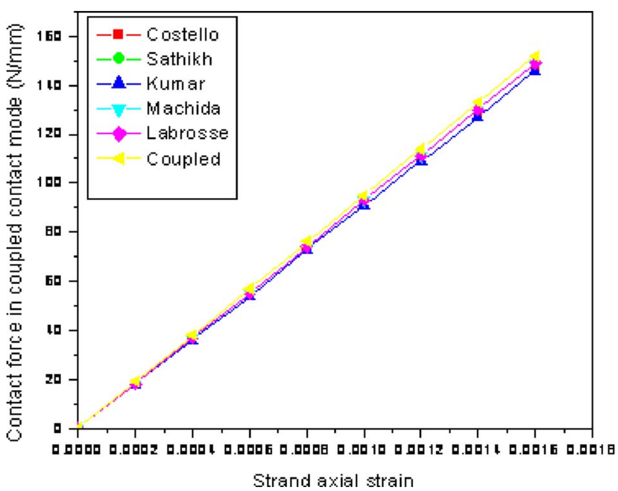

Fig. 8 Contact force

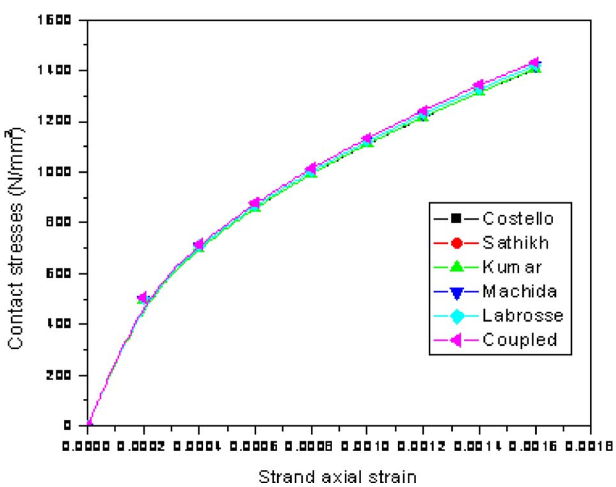

Fig. 9 Contact stresses

Nomenclature

C_f = contact force in normal direction (N/mm)
E = elastic modulus of core and helical wire $(\mathrm{N/mm^2})$
F_ε = axial stiffness coefficient due to axial load (N)
F_ϕ = axial stiffness coefficeint due to twisting moment (N mm)
F_c = axial force of the core (N)
F_w = axial force of the wire (N)
F = axial force of the strand (N)
G = normal bending moment of the wire (N mm)
G' = binormal bending moment of the wire, (N mm)
H = wire axial twisting moment (N mm)
h_0 = initial strand length (mm)
K = distributed wire unit moment about normal axis of wire (N mm)
K' = distributed wire unit moment about binormal axis of wire (N mm)
l_0 = initial length of the helical wire (mm)
M_w = wire moment about strand axial direction (N mm)
M_ε = twisting stiffness coefficient due to axial load (N mm)
M_ϕ = twisting stiffness coefficient due to twisting moment $(\mathrm{N\ mm^2})$
m_c = twisting moment of the core (N mm)
M = strand twisting moment (N mm)
m = number of helical wires
N, N' = wire normal and binormal forces (N)
p_0, p = initial and final pitch of the strand (mm)
P, Q = tangential distributed forces between core-wire (N/mm)
q = lateral pressure $(\mathrm{N/mm^2})$
R_{w0}, R_w = underformed and deformed radius of helical wire (mm)
R_{c0}, R_c = undeformed and deformed radius of core, (mm)
R = mean radius of strand (mm)
r_0, r = initial and final helix radius (mm)
S = normal distributed force between core to wire, (N/mm)
T = axial force of the wire (N)
U = normal distributed force between wire to wire (N/mm)
V, W = tangential distributed forces between wire to wire (N/mm)
X, Y, Z = distributed wire unit force in wire normal, binormal and axial direction (N/mm)
Θ = distributed wire unit moment in wire axial direction (N mm)
β_0, β = initial and final contact angle (deg)
$\tau_0, \tau, \Delta\tau$ = initial, final, and change in twist of the wire (rad/mm)
$\kappa_0', \kappa', \Delta\kappa'$ = initial, final, and change in binormal curvature (rad/mm)

$\kappa_0, \Delta\kappa$ = initial normal curvature and change in normal curvature (rad/mm)
$\alpha_0, \alpha, \Delta\alpha$ = initial and final and change in helix angle (deg)
μ = coefficient of friction
ν = Poisson's ratio
ε_{tc} = core transverse strain
τ_s = angle of twist per unit length of the strand (rad/mm)
ε = strand axial strain
ϕ = rotational strain of the strand
ρ_w = radius of curvature of wire (mm)
σ_{nc} = normal contact stress with friction
χ = angle of outer sweepout in a plane perpendicular to the axis of the strand (deg)
ξ = helical wire axial strain

Appendix

1 Strand Elastic Theory. An extensive review of research works and a critical examination of various technical approaches were conducted by Parthasarathy and Sathikh [10]. The formulation of contact angle, wire axial strain, change in binormal curvature, change in twist, wire force and moments, and equilibrium equation is given in this section. Figure 1 shows the configuration and cross section of a loaded simple straight strand. The strand initially consists of a straight cylindrical core of wire radius R_{c0} surrounded by m helical wires of wire radius R_{w0} at the initial helix angle α_0. The wires initially touch each other along m helical lines and touch the central core along another set of m helical lines. Hence, the initial radius of the helix of an outside wire is given by the expression (A1). Equation (A2) yields the radius of the wire helix in which the wires are just touching each other [4]:

$$r_0 = R_{c0} + R_{w0} \tag{A1}$$

$$r_0 = R_{w0}\sqrt{1 + \frac{\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m}\right)}{\sin^2 \alpha_0}} \tag{A2}$$

Three important conditions exist with respect to Eq. (A2).

(i) If helix radius is less than the right hand side of Eq. (A2), then the wire-wire contact exist.
(ii) If helix radius is greater than the right hand side of Eq. (A2), then the core-wire contact exist.
(iii) If the condition as given in Eq. (A2) is satisfied, the outer wires will touch each other and also with the central wire simultaneously.

The contact angle can be determined from the intersection of the projection of the cross section and the projection of the line of contact. It is found that

$$\cos \beta_0 = \frac{1}{\cos^2 \alpha_0}\left[\sqrt{1 + \frac{\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m}\right)}{\sin^2 \alpha_0}} - \sqrt{\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m}\right)\left(1 + \frac{1}{\tan^2 \alpha_0 \cos^2\left(\frac{\pi}{2} - \frac{\pi}{m}\right)\left(\sin^2 \alpha_0 + \tan^2\left(\frac{\pi}{2} - \frac{\pi}{m}\right)\right)}\right) + \sin^4 \alpha_0}\right] \tag{A3}$$

The principal normal curvature, binormal curvature, and twist of the center line of the undeformed wire are

$$\kappa_0 = 0 \tag{A4}$$

$$\kappa_0' = \frac{\cos^2 \alpha_0}{r_0} \tag{A5}$$

$$\tau_0 = \frac{\cos \alpha_0 \sin \alpha_0}{r_0} \tag{A6}$$

The pitch, height, and radial distance of the undeformed strand from Fig. 1(*b*) are found to be

$$\tan \alpha_0 = \frac{p_0}{2\pi r_0} \tag{A7}$$

$$h_0 = l_0 \sin \alpha_0 \tag{A8}$$

$$\chi_0 r_0 = \frac{h_0}{\tan \alpha_0} \tag{A9}$$

The change in helix angle from the geometry

$$|\Delta\alpha| = |\alpha\text{-}\alpha_0| \ll 1 \tag{A10}$$

The strand axial strain is

$$\varepsilon = \frac{h\text{-}h_0}{h_0} \tag{A11}$$

The strand strain can also be also be arrived from the wire axial strain and change in helix angle as under

$$\varepsilon = \xi + \frac{\Delta\alpha}{\tan \alpha_0} \tag{A12}$$

The strand rotational strain is given by

$$\phi = \tau_s R = \frac{\xi}{\tan \alpha_0} - \Delta\alpha + \frac{\nu[(\nu R_{c0}) - (\nu R_{w0})]}{r_0 \tan \alpha_0} \tag{A13}$$

where "R" is the mean radius of strand.

The deformed core, wire radius, helix radius, helix angle, change in binormal curvature, and change in twist are expressed by the following relations:

$$R_c = R_{c0}(1 - \nu\varepsilon) \tag{A14}$$

$$R_w = R_{w0}(1 - \nu\xi) \tag{A15}$$

$$r = R_c + R_w \tag{A16}$$

$$\alpha = \alpha_0 + \Delta\alpha \tag{A17}$$

$$\Delta\kappa' = \frac{\cos^2 \alpha}{r} - \frac{\cos^2 \alpha_0}{r_0} \tag{A18}$$

$$\Delta\tau = \frac{\cos\alpha\sin\alpha}{r} - \frac{\cos\alpha_0\sin\alpha_0}{r_0} \quad \text{(A19)}$$

2 Wire Equilibrium Equations. The six equations of equilibrium of a twisted and curved slender rod [11] are given by

$$\frac{dN}{ds} - N\tau + T\kappa' + X = 0 \quad \text{(A20)}$$

$$\frac{dN'}{ds} - T\kappa + N\tau + Y = 0 \quad \text{(A21)}$$

$$\frac{dT}{ds} - N\kappa' + N'\kappa + Z = 0 \quad \text{(A22)}$$

$$\frac{dG}{ds} - G'\tau + H\kappa' - N' + K = 0 \quad \text{(A23)}$$

$$\frac{dG'}{ds} - H\kappa + G\tau + N + K' = 0 \quad \text{(A24)}$$

$$\frac{dH}{ds} - G\kappa' + G'\kappa + \Theta = 0 \quad \text{(A25)}$$

The bending moment in the normal direction, the bending moment in the binormal direction, the twisting moment, and the tension of the wire are found by the following relations:

$$G' = \frac{E\pi R_{w0}^4}{4}\Delta\kappa' \quad \text{(A26)}$$

$$H = \frac{E\pi R_{w0}^4}{4(1+\nu)}\Delta\tau \quad \text{(A27)}$$

$$T = E\pi R_{w0}^2\xi \quad \text{(A28)}$$

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